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CONTINUA AND DISCONTINUA.

AS a first example of a continuum for perception take smooth and even shading from the very dark grey we call black into the very light grey we call white. Assume that this can be adequately presented to observation, say on a strip of paper, all black for some little distance on the one hand, all white for some little distance on the other hand, with continuous shading between. Just where, on such a strip of paper, lie the ends of the span of shading in which there is continuous change of quality of grey, it is very difficult, indeed practically impossible, to say. The exact position of the end on either hand eludes the imperfect delicacy of our powers of perception. We can however from observation say that 'here' the paper is still undoubtedly black and that 'there' it is just perceivably lighter than black. We may therefore infer that the end of shading is somewhere between the 'here' and the 'there,' and is probably somewhere in the mid-region between the two. We must take the ends of shading, then, as only approximately assignable. Let us now snip off from our slip so much, just external to 'there,' as to leave only a central portion within which we feel sure that there is continuous change of quality in the grey from darker to lighter.

Note that we have now for perception quite definite and readily visible ends—those imposed by our cuts on either hand. Here there is discontinuity of our excised strip with that which lies beyond it. But within the span which we have thus cut out or isolated, there are no visible ends at which it is divided into parts. In this sense it is a partless whole presenting continuously and uniformly shading qualities of grey. But by saying that there are no parts within the whole I do not mean that it is impossible to distinguish, say, the first inch as different from the third inch. No doubt we can by acts of attention isolate such parts for comparison. But then we are imposing ends to the parts which we so isolate by attention. I mean, therefore, that within the continuous whole there are no parts, given as such, prior to some process of isolation. I mean, too, that there are no such parts for perception. There may be invisible ends, and parts defined by them, which elude our powers of perception. But that is another matter.

Although there are, in the above sense, no given parts with visible ends *within* the continuous whole for perception, it is obvious that we can none the less cut out parts *from* the whole. By a part of a continuum we shall therefore mean a part isolated in some way from the whole. We must remember however that such an isolated part is, in strictness, an entity distinct from the whole. Thus we may cut out from our strip of shading a part for comparison with a whole similar to that from which it was isolated. It too has ends where the cuts were made; it too is a continuum; and it too affords to perception uniform continuity of shading—unless, as we shall presently have occasion to see, too small a part has been cut out. Leaving this on one side, we may say that any isolated part is *like* any other part, and is like the whole, or any like whole, from which it was isolated. By 'like' we are here to understand having observed similarity in the fact of continuous and uniform shading, though the range of such shading differs. On this understanding we may say that in the case of such a perceived continuum *the isolated parts are like each other and like the whole from which they were isolated.*

But we need not actually cut out an isolated part, say with a pair of scissors. By viewing our strip of paper through a suitable tube of small bore we can isolate just that part which is within the field of vision. At nearer distances the isolated parts are progressively smaller; at further distances they are progressively larger; by moving the tube laterally we can isolate a part anywhere along the whole length of our strip. We may however discard both scissors and tube, and we may imagine cuts to be made here, there, or elsewhere. Such an ideal but not actual cut we will call a *defining cut,* since it defines the positions at which limits of isolated parts may be imposed. And we will assume that such a defining cut may be made anywhere at right angles to the direction of continuous shading. Then, as before, any large enough part, limited by two defining cuts, will be like the whole. There will be similarity in that each presents continuous and uniform shading from darker to lighter.

Take next a segment of a circle. For our acquaintance under careful observation it has uniform and continuous curvature. Within its span the segment is for perception an undivided whole, and has no perceivable parts with visible ends. But by actual cuts, or by the use of a tube, or by defining cuts, we may in practise, or for thought, isolate a part; any such part is like the whole or any like whole from which it is isolated, and is like any other part so isolated. Whole and parts, if the parts be large enough, have for perception likeness of continuous and uniform change of direction. The question here arises whether the fact that we can, say by two defining cuts, isolate a part anywhere, justifies us in asserting that, prior to some such isolating process, the whole is made up of parts. Are we, by our isolating procedure, *discovering* parts already pre-existing in the continuous whole? Or are we *making* such parts for purposes of interpretation? I submit that by isolating them we make them, *quâ* parts. I submit that perception is unable to discover parts until manipulation or thought imposes cuts actual or ideal.

It may perhaps be said that at any rate the distance between any two cuts is infinitely divisible. Of course when we speak of infinite divisibility we are going far beyond perception—but let that pass. It is moreover a tenable position that no given distance is divisible into lesser distances, save under a convention. Any given distance is just what it is—the spatial relatedness between two positions as terms. We may compare it with other distances. But it is not composed or made up of lesser distances. The point is however a subtle and debatable one: so let that too pass. What then exactly do we mean by saying that a line is infinitely divisible? Do we mean that, in such a line, between any two cuts, a and c, a third cut b may be made anywhere; similarly between a and b, between b and c, and so on? If so, it may be granted that such cuts can ideally be made. But does that show that the continuous line was already divided before the cuts were imposed? If not, what does it show? It shows that there is in the line, prior to cutting, a continuous order of occupied positions, some one of which will be found by any defining cut. But does not this continuous order of occupied positions just bring us back to the continuum with which we started—now tacitly correlated with a continuous order of real numbers? Is any justification given for regarding the line as *ab initio* composed of parts which are themselves little lines, placed end to end? If it be said that it is composed, not of little lines but of parts which are points in a continuous order, the answer is that this continuous order implies relatedness of points, and the spatial relatedness of points in the order under consideration brings us back to a line. In any case, for perception, the appearance with which we are dealing is unquestionably that of a partless whole.

Suppose now we observe the swing of a pendulum through an arc, say from left to right. Here the space traversed—the line traced by the pendulum-bob—presents nothing new for consideration. Any cut therein defines a position; any two cuts define a spacial part which is thus isolated from the whole. But change of position involves time-relatedness. In the case of motion any cut defines not only a space-position but also a time-instant; and any two cuts define an interval in the order of time-relatedness. Can we be said to perceive such a time-interval? At any rate we perceive the motion and can proceed to interpret what we perceive as involving time-relatedness. We can observe the swinging pendulum through our tube and thus isolate a part from the swing as a whole. As the bob enters and leaves the field of vision restricted by the tube, we have the beginning and ending of the part of the swing thus isolated. No doubt the isolated part may fall entirely within the perceptual present; but within that present we are directly acquainted with relations of before and after; and this affords the perceptual basis for time-interpretation. By suitably adjusting our tube we may isolate a part of the swing anywhere for independent observation and for comparison with other parts isolated in like manner, or with the whole swing from left to right. Or, discarding the tube we can isolate a part of the swing by purely ideal cuts, each of which defines a position in space occupied at an instant of time. Dealing thus with motion 'in space and time' (as we commonly say) it is important to note that any part isolated from a whole of motion, is itself motion; and that if the whole motion is continuous, then is any isolated part thereof a continuum. But it is not uniform in the sense in which we have used that word. The velocity changes in the course of the swing from left to right, increasing to a maximum, then diminishing to the end-point at which motion ceases. But the change of the rate of change of position seems to be none the less continuous. Not all continua therefore are uniform continua in which the isolated parts are all alike.

No doubt it will be said by philosophers of the Bergsonian school that if we deal with time by the method of cuts we are treating it spatially; by which expression I understand them to mean in correlation with the fixed order of number which is applicable also in our dealing with space. They rightly insist on the reality of change, in which alone, they urge there is true continuity. But with change they identify time—that is, what they call real time (duration) as contrasted with fictitious time as a mere schematic device of the cold intellect which freezes that which in its fluid flow is felt in the genial warmth of intuition. For intuition, I take it, there are neither terms nor relations; there is just change such as we enjoy when the life within us glows to consciousness. It is life which is both change and source of change for a philosophy in which all intellectual distinctions melt and merge. We too insist on the reality of change, the continuity or discontinuity of which is open to discussion. But for us change is only interpretable as change of relatedness of terms. Instants, as the terms for time-interpretation, may be determined by cuts through the flow of events—which for us is *not* the flow of time. Such cuts are, no doubt, an intellectual device; no doubt with a little lack of imagination they may be regarded as inevitably spatial. But in no sense do they stop the given flow of change; and in no time sense do they yield points of arrest in that flow, any more than a snap-shot photograph of a swinging pendulum arrests its motion. We contend that if change and time be identified, then one or the other is merely a redundant synonym. Let us in that case deal with what is given either in terms of change or in synonymous terms of time, or, if we can, perhaps in the absence of terms or relations. For it is clear that no relation, other than verbal, obtains between different names for the same entity. Cuts then, for us, are methodological devices for dealing with certain real relations spatial or temporal between phases of change—relations that are wholly independent of the methodological device by the help of which they are interpreted.

Now I said, some little way back, that, when a uniform continuum is presented for observation, parts isolated therefrom are like each other and are like the whole from which they are isolated. But, in leading up to this statement, I was forced on observational grounds to introduce as a qualification "unless the isolated parts be too small." The grounds on which this qualification must be introduced are tolerably obvious. On viewing, for example, a large segment of a circle of, say, ten-foot radius, we may assert with some confidence that the visible line has uniform curvature. Of course in such a case judgment is involved; but it is judgment on observed matter of fact presented in the field of perception. If however a very short part of the whole segment be isolated, it may be indistinguishable from a straight line. Its curvature is so slight as to be imperceptible. And since it is imperceptible, there is no ground in direct observation for asserting its presence. In its absence the small line is judged to be straight. Here therefore the small part is not judged to be like the whole from which it was isolated. But if we happen to know that we are viewing an isolated part of a large circle, we may perhaps assert that, although the small line is *apparently* straight, it is *really* very slightly curved. From the apparent curvature in the whole we infer real but not apparent curvature in the part. And very likely, from a consideration of this and other similar cases in which our powers of perception are not acute enough to form the basis of a true judgment without going beyond that which we then and there perceive, very likely we may go further and assert as a general truth that in case of really uniform continuity any part isolated by the method of cuts, no matter how small, is really like any other part, and like the whole from which it was isolated; and that this will hold good whether the likeness in question be perceivable by us or not. Now what do we imply here by the use of the word 'real?' First, I take it we imply, in this use of the word, that there is a knowable object to which that which we speak of as perceivable more or less closely corresponds; and secondly I think we imply that the likeness, imperceivable by us, would be perceivable by a being whose powers of perception were trained to a sufficiently high pitch of delicacy. We seem also to imply, in such a case, that the knowable object as a whole is given in our perception as it really is in some particular respect, though this may not hold good of its small isolated parts, since, through lack of delicacy of perception, we are not prepared to say whether appearance closely corresponds to reality or not—that is, to that reality, for common sense and science, of which we are in quest. But suppose that what is taken to be the segment of a large circle is really, as we say, a regular polygon with very many sides, each a millimeter in length. This might well be for our perception indistinguishable from a segment of a circle. If however a photograph of a small part were examined under a microscope, our aided perception might well reveal in the part the real polygonal figure. In this case we should probably infer from the nature of the isolated part, as revealed to our aided perception, that the whole was of like nature, though our perception of the whole failed to reveal its polygonal character. As a matter of scientific procedure we sometimes infer from the whole to an isolated part, and sometimes from an isolated part to the whole. We take our departure from that in dealing with which our perception is at its best, and from this we infer reality of like nature where the conditions of observation are less favorable. But with sharpened powers of perception the knowable object would throughout its range appear as it really is in that respect which is under examination. I suggest, then, that by what is really continuous or discontinuous we should understand that which would appear to be such to a being whose delicacy of perception was sufficiently refined. Such reality would still be appearance, but appearance to enhanced percipience as a basis for judgments. It should be noted that we here frankly start from perception, and that what we propose to speak of as real, though it may involve conceptual thought in its attainment, is still within the same perceptual mode. Stress should be laid on this restriction to the same perceptual mode. If we are dealing with discrimination based on our perception of musical pitch, enhanced delicacy would still be in the perception of musical pitch. We must not jump from audible sounds to the vibrations with which they may be correlated—even if we feel justified in supposing that to a being of indefinitely heightened acuity of perception these might be perceived in some *other* manner than by hearing. To perceive vibratory motion would involve a different mode of perception from that of being acquainted with musical pitch of audible tone. No doubt the knowable object, in the fulness of its nature, is knowable in many different ways, through different channels of perception, and by different avenues of inference. But what I have suggested as that which we commonly mean when we say that this is really so, though we fail so to perceive it, must be restricted to the mode of perception in question; otherwise it is not, I think, what we mean when, in such phrases, we speak of appearance and reality. It is on this understanding that I submit that, for a being whose acuity of perception was thus sufficiently sharpened, it would be an observable fact that this or that was continuous or discontinuous in any given case. It is scarcely necessary to say that not all cases of continuity or discontinuity can be dealt with thus. Perceiving, thinking, enjoying, and all conscious processes, as such, are wholly intractable to this method of treatment. None of these is perceivable in the sense in which the word is here used.

Let us now follow up a little further the continuity that is presented to perception. Revert to continuous and uniform shading—for example, to vary the illustration, that from red into yellow through orange. Isolate sufficiently small parts in serial order and label them with the letters of the alphabet. Viewed through a tube at appropriate distance, to be determined experimentally, no such isolated part from *a* to *z* is distinguishable in colors from its neighbors; and each part is, throughout, of one and only one color. None the less the color of *a* is readily distinguishable from that of *m*; and *m* from *z*. The isolated part *a* is red; *m* is orange; *z* is yellow. But in accordance with what has been suggested above, it seems permissible to infer from the apparent continuity of shading in the whole that there is real shading in *a*, or any other similarly isolated part, though our powers of color-discrimination, even after careful training, are not such as to enable us to detect it.

With suitable rate of shading let *a* be indistinguishable from *b*, *b* from *c*, and *c* from *d*. In each case the isolated parts *a*, *b*, *c*, *d*, are apparently self-colored. No difference of color is perceivable within any one of these isolated parts. But now enlarge the area isolated within any part by reviewing the shaded whole from a greater distance. A stage is reached at which within such a more extended part there is just perceivable difference of color. The experimental work is not easy, since the difference in question is only just perceivable; but, as far as it goes, the results of observation seem to point pretty definitely to the conclusion that, if the shading of the whole be apparently uniform and continuous, what we may speak of as the distance for just perceivable difference, is the same *anywhere* in the whole range of continuous shading; and that an indefinite number of isolated parts with just perceivable difference are not only alike in this aspect, but are also alike in that the cuts which isolate the parts are equidistant. The inference seems to be that the just-perceivable-difference-distance is a measure of the percipient's delicacy of perception; and that within the parts *a*, *b*, *c*, *d*, in which there is no perceivable difference, that measure is not reached; but that there is in these parts a real difference—one, that is, which would be discriminated by a percipient whose delicacy of perception was sufficiently increased.

This inference may be supported if we approach the problem in a somewhat different way. Shading that has the appearance of being continuous and uniform from yellow through orange to red, may be obtained on a rotating disc. On a cardboard disc is pasted red paper which covers the whole surface. Over the red paper are pasted, say, three smooth-edged leaves of yellow paper. Where the bases of these leaves meet there is unbroken yellow; just beyond their tips is unbroken red; between these limits there is a smoothly decreasing percentage of yellow. Experimental work shows that, in accordance with Weber's Law, the increments of yellow at equidistant stages from tip to base are in G. P. This, at any rate, with a factor experimentally determined, gives apparently continuous and uniform shading, from red to yellow, when the disc is rapidly rotated. Now viewed through a tube, a small enough isolated part of the rotating disc is apparently self-colored. No color difference can be detected in the area within the restricted field of vision. None the less there is a different proportion of yellow to red near one limit from that which obtains near the other limit. There really is, for measurement on the still disc, a difference which is easily determined; but there is no apparent difference when the disc is rotating. It seems then that under rotation we cannot perceive difference which measurement, under other conditions, shows to be really true. Such observations lend some support to the reference drawn in the last paragraph.

Consider now the much-discussed 'Stumpf paradox.' Let a', b', c', d', be colored wools matching the a, b, c, d, of continuous shading. Then a' is indistinguishable from b', b' from c', and c' from d'. But a' is distinguishable from c', and b' from d'. It seems then that, since b' is indistinguishable from both a' and c', it can have no status in the range of *our* perception. It has, however, *inferentially*, a status of its own, even in our perception. For though b' is indistiguishable from c', it is distinguishable from d'. It is thus differentiated from c'. Similarly though b' is indistinguishable from a', still a' is distinguishable from c'. Hence b' is also differentiated from a'. And if b' is thus differentiated both from a' and from c' it has inferentially a status of its own, even in our perception. On these grounds it seems that b' lies *between* a' and c' in a color order, although we cannot directly perceive its difference from a', or its difference from c'. And it seems to be legitimate to suppose that, to a percipient with more delicate powers of perception, b' would be directly, and not only inferentially, discriminated. For him there would be inferentially intermediate steps in the color order, between a' and b', b' and c', and c' and d', not directly perceived as different from their neighbors, but such as would be discriminated by a percipient with yet more delicate powers of perception. For that percipient in turn there would be inferred intermediates, distinguishable to a higher order of perception, and so on indefinitely.

It will here be helpful to attempt some correlation with number; in which I hope that I may not fall into error. Take our colored wools; they may be correlated with integers; a' corresponds to 1, b' to 2, and so on. But if we intercalate a wool intermediate in color between a' and b' in the manner suggested in the last paragraph, there is no integer with which to correlate this intercalated wool. It may be urged that we can correlate it with the fractional number 3/2; but this number is not an integer; it is a *rational* which belongs to a logically distinct type of number constructed by taking pairs of integers according to an accepted principle. Since however this principle of construction gives us *inter alia* rationals of the form $r/1$ we have a homogeneous series in which $r/1$ may be correlated with r in the series of integers. But in practice we substitute the integer, r, for its correlated rational $r/1$, and deal, by a recognized convention, with a heterogeneous series of integers and rationals.

Given the homogeneous series of rationals, we can correlate an intercalated rational with any intercalated member of our imaginary series of wools. But we want to go further than this. We want to correlate a number of some number type, with any defining cut in a *continuous* order. For this the type of rationals will not serve our purpose. Suppose for example we are dealing with a straight line which we assume to be really continuous, and suppose our defining cut is such as to isolate a part which bears to the whole line the relation which the side of a square bears to its diagonal. There is no rational which corresponds to the position of our defining cut. To define its position, in correlation with number, we require a new (and again logically distinct) type of number constructed on a different principle. Such a type is that of the *real numbers*. This type includes members which may be correlated with the rationals, as the rationals include members which may be correlated with the integers; but it also includes members which cannot be so correlated; and all its members form a homogeneous series. It meets the case of the occurrence of such an *irrational* as $\sqrt{2}$.

The principle of construction does not here specially concern us. But it may be profitable for us briefly to consider the correlation of real numbers with cuts in our continua. Let a defining cut be made anywhere within the segment of a circle. What it defines is a *limit* in that which I understand to be the technical sense of the term. This limit is susceptible of correlation with a real number; and it may also be susceptible of correlation with a rational. To the left of the cut there is a continuous sub-segment S', and to the right there is a continuous sub-segment S''. So too in the correlated number scheme; to the left of the limit is the whole infinite collection of rationals below the limit; to the right the whole infinite collection of rationals above the limit. But the limit itself falls within neither the one nor the other; it can not be comprised in that which is either to the left or the right of itself, nor in that collection of rationals which is either below or above itself. The defining cut marks a limit which has its own quite definite position in an order. But that which is the upper limit of what goes before is also the lower limit of what comes after. Thus $r/1$ is the upper limit of the whole collection of rationals below $r/1$, and the lower limit of the whole collection of rationals above $r/1$, but is not itself included in either collection.

A defining cut thus gives unambiguously a position in the segment of our circle which is the limit of sub-segments S' and S'', say to left and right, as parts thus isolated from the whole uncut segment; and this position may be correlated with a real number; at any rate (as I understand) positions which cannot be so correlated have not yet given occasion for a fourth species of the genus number. Assuming that real numbers suffice to meet all cases, and assuming that apparent continuity, say of motion, is really continuous, we may not only say that any isolated part of a uniform continuum is like any other part in like manner isolated, and is like the whole from which, as part, it is isolated, being also like any part isolated from a like whole; we may say further that it is *equivalent to* any other part or to the limited whole.

The sense in which this may be said may be illustrated through reference to a suitable modification of the old problem of Achilles and the tortoise. Assume that the representatives of Achilles, A, and the tortoise, T, are in continuous sliding motion, the rate of change of position bring in each case constant. Assume that in 10 seconds, A slides 100 yards, and that T slides 10 yards. Let T have 90 yards start before either of them slides. It is clear that when A has covered 90 yards in 9 seconds T has covered 9 yards; and that when A has progressed through the succeeding 9 yards in .9 of a second, T has moved on .9 of a yard. And it is also clear that by the method of adding progressively diminishing rational increments we can never reach the terminal limit of 100 yards for A, 10 yards for T, and 10 seconds for both A and T. We can only reach $99.\dot{9}$ yards for A, $9.\dot{9}$ yards for T in $9.\dot{9}$ seconds. In this pretty obvious sense A is never abreast of T under these additive conditions. But now assume, on the basis of observations in analogous cases, that A *does* get abreast of T at the limit of their several lines of progress, which is also the limit of the span of time in which the feat is accomplished. The limit of A's slide and of T's slide correspond. Make now a time-cut *anywhere* before the limit of 10 seconds. That cut will also be a defining cut marking A's position in his course and T's position in its course. These positions correspond *inter se;* they correspond to a determinate instant in the time-span; and they correspond perhaps to some member in the order of rationals. Suppose the temporal defining cut is at half-time, the positions of A and of T are half-space. There is one-to-one correspondence to a rational with which the cuts are correlated. So too there is one-to-one correlation with 4/5, 9/10 and so on. And if a defining cut, made anywhere, chances to miss a position which can be correlated with a rational, it none the less defines a position which, on the assumption previously made, can be correlated with an irrational which will be named and symbolized according to our need in each case that arises. Throughout, therefore, there is one-to-one correspondence in the technical sense of this phrase. A member of some species of the genus number is ready to hand if it be needed. And where cuts common to 'this' and 'that' give one-to-one correspondence, 'this' is said to be *equivalent* to 'that.' It remains only, in this connection, to notice that we may compare an isolated part of A's course between limiting cuts, with A's whole course. In this case, too, there is one-to-one correspondence. And in this sense it may be said that the part of a continuum which is what we have called like the whole, is also equivalent to the whole—though it should not be inferred from this statement that it applies to continua only.

I have dealt, for convenience of treatment, almost exclusively with uniform continua, and have essayed to pass from the apparently continuous for perception to the really continuous, using the word 'really' in a sense that does not involve any divorce from 'apparently.' By a uniform continuum I mean one in which any part, isolated by cuts, is like any other part and like the whole from which it is derived—like, that is to say, in some definite character which is selected for investigators. Thus a straight line has continuous and uniform direction. In a segment of a circle there is continuous and uniform change of direction. In any part of a parabolic curve there is continuous and uniform change of change of direction. In any of these cases the law of the part is the law of any other part or of the whole. But a curve of indefinitely varying direction is continuous but not uniform in the sense in which I am using the word. The law of any selected part is not necessarily the law of any other part; nor is it the law of the whole.

Now what is characteristic of any perceived or perceivable continuum, whether uniform or not, is that it presents a *slide* and not a series of *steps* however minute. If the motion of a pendulum-bob be really continuous, its center slides through the arc of a circle without pause or jump. Let us assume, for the purpose of interpreting this motion, a continuous order of positions and a continuous order of instants. Then the center slides along each of these orders. It slides, as we say, through positions in space, it also slides, as I think we ought to say, through the order of instants in time. Each position in space is occupied at an instant of time—such occupancy to be of course clearly distinguished from *rest*, which is the occupancy of *one* position in correlation with *more than one* instant. Every defining cut defines unambiguously a position in the one order, and an instant in the other order, each of which has its own unique individuality in the sense that it may be correlated with a real number which is just itself and nothing else without hint of merging or smudginess. Two cuts define two positions and two instants within the several relational orders. The relation of position to position and of instant to instant can thus be accurately stated in correlation with the relation of real number to real number. But the cuts must define and fix for our thought the unambiguously unique individuals which limit the isolated parts we thus make; only thus can they be rendered susceptible of exact treatment. Although, however, the relations in space and time are thus treated, and must be thus treated, statically, this is not a static treatment of motion. For motion is change; and to speak of a static treatment of change is a contradiction in terms. It seems then that we ought to distinguish the sliding which is the essential characteristic of all continuous change, from the continuous order or orders along which it slides—just as we distinguish the sliding boy from the slide along which he is sliding.

The objection may here be raised that, though he is moving through a certain order of positions along a plane of ice relatively fixed in space, it is absurd to say that he is also moving through a certain order of instants in time, if it be implied that this order is in any sense fixed. The absurdity, it may be urged, becomes obvious when we consider what happens when the boy comes to rest. How can we say, as was said above, that there is then a correlation of one position with more than one instant, if there be no flow of time to give a succession of instants during so many of which he occupies, or his center of figure occupies, one position? Well, question for question. Why should he not occupy one group of positions in space and forge ahead steadily along the continuous order of time? Why should he not be stationary in space and moving onwards in time? Why should there not, in other words, be change in time-relatedness but no change in space-relatedness? I see no absurdity here, if once we realize that what we speak of as existence endures through time, and that an ideal cut only gives an instantaneous view of it as it passes on, nowise arresting its passage. What endures is, among other things, unchanging relatedness in space; but not only this; all the other modes of relatedness by which its existence is defined. Still less absurdity is there if we also realize, as science has taught us to realize, the ubiquity of change. The stationary boy is a theatre of intricately interwoven, ever changing process. At no instant does this process cease. There is no object that is not the theatre of a moving drama of world-process. Even the stability of the atom is that of balanced change. From this point of view it is, in all cases, *process*—what we speak of as the course of events—which flows along the order of time; and it flows through so many instants of time, whether the center of the boy's bodily figure occupies one position in space or is sliding along an order of positions.

We need not however restrict our attention to process of which the boy's body is the theatre; indeed it is seldom that in the course of ordinary experience we pay any attention thereto. But when once we realize that the whole world-process is sliding or stepping onwards through the continuous order of time, we see the justification for our normal mode of procedure, which is to take for reference whatever process will best serve our purpose. We may find it convenient to correlate duration of rest with so much onward stepping of the hands of our watch, or with the amount of apparent onward movement of the sun in the sky or its shadow on a dial; or, failing then, much more commonly, with the flow of conscious process of which we have enjoyment. Unreliable as this last is, for scientific purposes, it is always at hand for reference, and it remains the primary and primitive basis of such reference. During our waking life we are continually, and perhaps continuously, passing onwards through the order of time, and with this onward passage we habitually, though not explicably, correlate the period of rest in a stationary object. What we here speak of as rest in space is analogous to that of the sleeping top in which changes occur in relation to a central position which preserves unchanged its relations to surrounding positions similarly occupied. And the period of its rest is the period of its onward passage through time as a specific order of relations. We are aware that it remains at rest through so much progress of our conscious life. But the changes within it keep pace with the changing universe, with which our conscious process, as a mode of change therein, keeps pace. So far as we know, nothing remains unchanged in relation to the order of time, though spatial centers around which these changes are grouped in the material world may either change or not change their relations to other centers in the order of space. In reference therefore to the spatial world our perceptive process slides or steps along that which we perceive; and what we call the permanence of objects is based on the continuous or discontinuous repetition of our percepts as our attention clings to them or reverts to them. Let us call this *perceptual travelling*, and remember that it is through such perceptual travelling over our objective world, following its relative movements, dwelling on its relatively quiescent phases, and moving along its relatively stationary lines, that we come to know the facts of that world, and hence to grasp its truths.

Suppose then that we are perceptually sliding along the segment of a circle wherein the uniform change of direction is really continuous. We cannot truly say that 'now,' and 'now,' and 'now,' there is change of direction in that along which we slide, if by this we mean that between these 'nows' the direction is unchanging. But that is just what we can truly say if we are preceptually travelling along the successive sides of a large enough part of a regular polygonal figure. Here, instead of continuous and uniform change of direction we have a discontinuous and rhythmic series of changes of direction. Between certain limits, however, those defined by cuts at the several angles, we are perceptually sliding in a continuous and uniform direction along a straight line. We have then in such a case,—so far as direction is concerned—a discontinuous series of continua, certain defining cuts marking the limits of these continua. But together with discontinuity quâ direction there is the strictest continuity of occupied positions. Suppose now that having drawn very faintly in pencil a regular polygon, and having marked with little ink dots the approximate positions of the limits defined by cuts at the angles,—we then rub out our faint pencil lines, leaving only the ink dots. There are now no continuous straight lines to be actually perceived. None the less, if we are imagining or thinking of a regular polygon, an order of relatedness of the dots sufficiently guides the perceptual sliding. But if a regular polygon be not in our thought, the dots may just as well define the positions of the sharp angles of a many-rayed star, or cuts on the segment of a circle, and so on. In each case the route of imaginary travelling differs. The important thing to grasp is that our dots, or the cuts which they roughly represent, are set in some order of relatedness, which order must be defined in our thought, if the dots or cuts are to have definite significance. Now of course in our varied world there are many cases of what seem to be disorderly discontinua. But they have comparatively little interest for scientific investigation. As therefore in considering continua we dealt with uniform continua, so now, in briefly considering discontinua, we will deal with *orderly discontinua;* and by an orderly discontinuum I here mean one in which any large enough part, isolated by cuts, affords the basis for a conclusion with regard to the form of any other such part, or of the whole from which it has been isolated.

Consider further the case of the regular polygon. Isolate small enough parts by tubular observation. Such a visible part may be (*a*) a straight line, or there may be enclosed (*b*) two straight lines which meet to form an angle. It is clear then that any given small part of the orderly discontinuum is not like every other small part, nor is it like the whole from which the small parts are derived by isolation. The given part will be like some other such parts, and will be unlike some other such parts. Again any such small part is unlike a larger part which includes, say, two angles, and so on.

Transfer attention now to a sufficiently large part. Such a part, though it may be like the whole from which it is isolated, in the same sense as is any part of a continuum, is none the less *representative* of that whole, in the sense that the law of the whole can be inferred from the adequate consideration of such a part. What is to be understood by 'sufficiently large' must depend on the conditions of the problem presented. I take it that in determining the figure of a regular polygon, the isolated part within which are comprised three straight lines and two angles is sufficiently large. From perceptually travelling along such a part, and I think no smaller part, the figure of the whole polygon, granted that it is regular and orderly, can be inferred. If the example I have chosen may be regarded as typical, we seem justified therefore in saying that an orderly discontinuum is one of which any large enough part, isolated by defining cuts, is representative of the whole, and that such a part affords a basis for a conclusion with regard to the form or law of the whole.

Now we have seen that in the case of uniform continua the isolated parts are made rather than found. We do not disarticulate parts already there; for the continuum is a partless whole. How stands it then with regular discontinua? In the polygon it may be said that the parts are straight lines each of a definite length; these parts are given as such; that our business is (1) to find them, and (2) to state the manner of their relatedness to each other. But on these terms we seem to divorce the part from the relatedness with which it must be wedded to constitute the whole. And, on these terms, from the part as such (*e. g.*, one side of a polygon) we can draw no conclusion as to the nature of the whole. It appears to me that we ought to distinguish an analysis into terms and relations from the isolation of representative parts for comparison with each other and with the whole—though such comparison of course involves terms and relations. What I venture to submit is that, in any logical discussion of whole and part, it would be better not to apply the word 'part' in such a case as the isolated side of a polygon. It would be better to call it a member of that collection of members which, in their specific mode of relatedness, constitute a representative part of a polygon. The point here is that any representative part of an orderly discontinuum gives not only constituent members but also their mode of relatedness. By the study of such parts we may reach valid conclusions with regard to the whole from which we isolate them. That, if I mistake not, is what, in effect, we do in the course of scientific procedure. Nature's wholes are often too large for our grasp; we therefore isolate representative parts from which we may infer the form and law of the whole. We may of course also infer from a larger to a smaller representative part; but we must not so disintegrate it that it ceases to be a representative part. Thus the chemical analysis of a compound into its elements does not yield representative parts. It yields only the dissociated members of what *was* a corporate body prior to such analysis.

I have here essayed to deal only with relatively simple types of continua and discontinua. I have dealt with them as they are presented in the world that lies open to our perception, or which would, as I conceive, lie open to the perception of a being with enhanced powers. I venture to think that only by starting thus can we hope to reach conclusions which may form a basis for the consideration of more complex cases. In the very brief discussion of orderly discontinua I have perhaps said enough to show that, by the isolation of representative parts for comparison with larger wholes, data may be reached which are of value for the purposes of science. But with the powers of perception which we possess, it is, I take it, extraordinarily difficult, if not impossible, to decide whether what is presented in any given case is really a very fine-grained discontinuum or a true continuum. All we can do is to follow up the quest as far as we can, and to be cautious in making assumptions beyond the range of our assured grasp. No doubt in view of my method of approach I shall be charged with perverse topsy-turvydom. It may be that, as we are so confidently told, the flow of our conscious life should afford our point of departure. It may be that this reveals to intuition an isolated part of the continuous push of the larger life with which the world is instinct, in spite of the apparent discontinuity introduced by the intellect. The part of which each one of us is conscious in its continuous simplicity may be like the greater whole—one and indivisible. It may be that thus, and thus only, is continuity known in its primal purity. Or it may not be so. Our conscious enjoyment may, not impossibly, afford the supreme example of pseudo-simplicity. Its seeming continuity may be analogous to that of a rope woven out of myriads of discontinuous fibres. Our complex life may exemplify the furthest known remove from continuity in its native purity. Who shall say? But such problems are beyond the modest scope of such considerations as have been outlined in the foregoing pages.

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